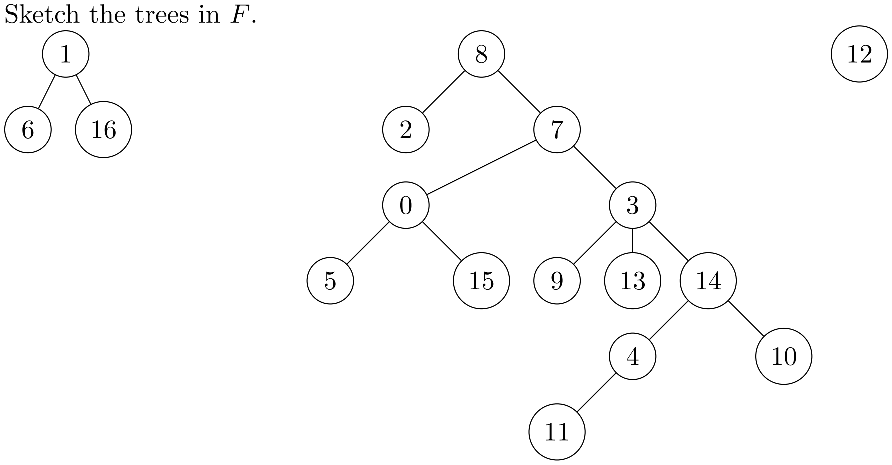
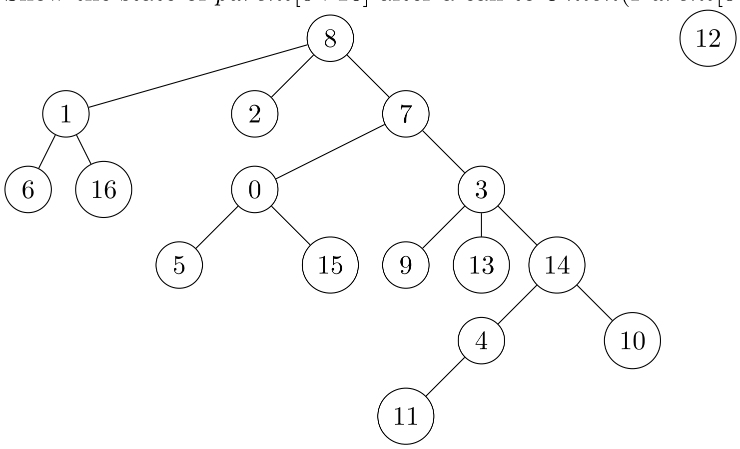
1

a. Sketch the trees in F.

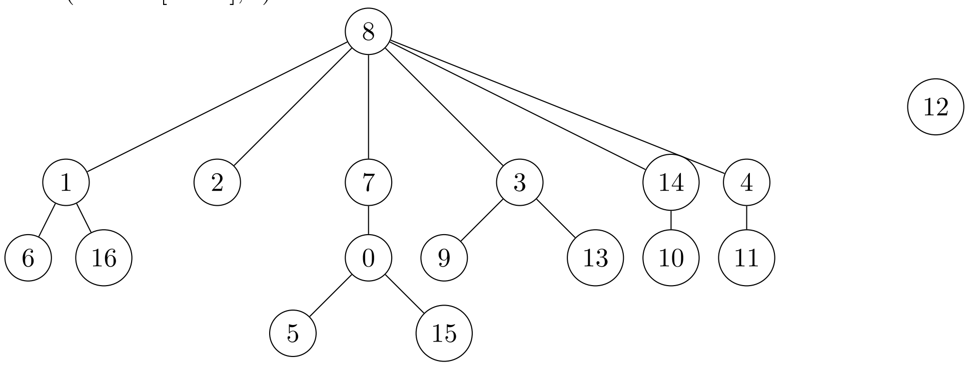


b. Show the state of $parent[0:16]$ after a call to $Union(Parent[0:16], 1, 8)$



i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
$Parent[i]$	**7**	**8**	**8**	**7**	**14**	**0**	**1**	**8**	**-16**	**3**	**14**	**4**	**-1**	**3**	**3**	**0**	**1**

c. Given the state of $Parent[0:16]$ in part (b), show the state of $Parent[0:16]$ after an invocation of $Find(Parent[0:16], 4)$ and sketch the trees in F.



i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$Parent[i]$	**7**	**8**	**8**	**7**	**14**	**0**	**1**	**8**	**-16**	**3**	**14**	**4**	**-1**	**3**	**3**	**0**	**1**

2 State and prove a generalization for k-ary trees:

a. Given that the depth of a complete binary tree T_n is given by $d(T_n) = \lfloor \log_2 n \rfloor$.

If T is a complete k-ary tree, then the number of nodes, n, we have

$$\begin{aligned} n &\leq 1 + k + \cdots + k^d \\ &= \frac{k^{d+1} - 1}{k - 1} \\ n(k-1) &\leq k^{d+1} \\ \log_k(n(k-1)) &\leq d + 1 \\ \log_k(n(k-1)) - 1 &\leq d \end{aligned}$$

And since d, the depth of the tree, is an integer, we can take the ceiling of that expression as our answer.

c. For any $k - tree$, we will have k leaves for each internal node. Thus, proposition 4.2.3 states for 2-trees that
$I(T) = L(T) - 1$
This could be equivalently written for a 2-tree, where $k = 2$
$I(T) = \frac{L(T)-1}{k-1} = \frac{L(T)-1}{1}$
That holds for the base case, and
$I(T) = \frac{L(T)-1}{k-1}$
holds in the general case of $k - trees$

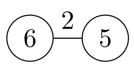
3

a. Trace the action of procedure *Kruskal* for *G*.
Implemented with Graph ADT, Priority Queue ADT, Forest ADT using Parent array, Collection of Disjoint Node sets.

Step 0: Initial Conditions

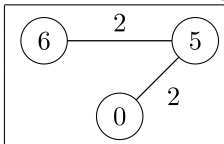
i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Parent[*i*]	**-1**	**-1**	**-1**	**-1**	**-1**	**-1**	**-1**	**-1**
Node sets	{0},{1},{2},{3},{4},{5},{6},{7}							
Selected edge	{}							

Step 1: Select edge 5-6 between Nodes



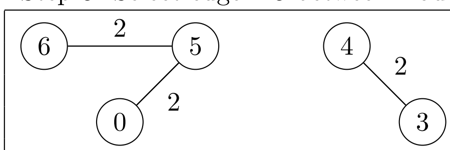
i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Parent[*i*]	**-1**	**-1**	**-1**	**-1**	**-1**	**-2**	**5**	**-1**
Node sets	{0},{1},{2},{3},{4},{5,6},{7}							
Selected edge	{5-6}							

Step 2: Select edge 0-5 between Nodes



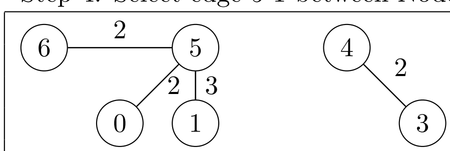
i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Parent[*i*]	**5**	**-1**	**-1**	**-1**	**-1**	**-3**	**5**	**-1**
Node sets	{1},{2},{3},{4},{0,5,6},{7}							
Selected edge	{0-5}							

Step 3: Select edge 4-3 between Nodes



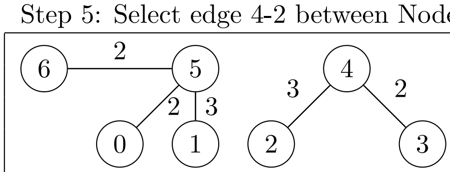
i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Parent[*i*]	**5**	**-1**	**-1**	**4**	**-2**	**-3**	**5**	**-1**
Node sets	{1},{2},{3,4},{0,5,6},{7}							
Selected edge	{4-3}							

Step 4: Select edge 5-1 between Nodes



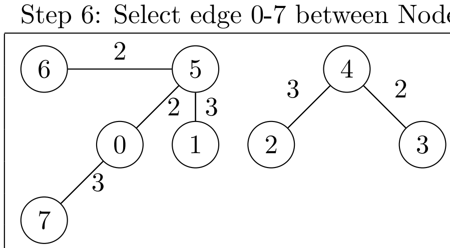
i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Parent[*i*]	**5**	**5**	**-1**	**4**	**-2**	**-4**	**5**	**-1**
Node sets	{2},{3,4},{0,1,5,6},{7}							
Selected edge	{5-1}							

Step 5: Select edge 4-2 between Nodes



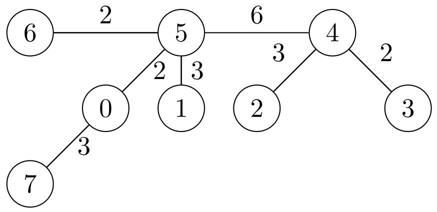
i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Parent[*i*]	**5**	**5**	**4**	**4**	**-3**	**-4**	**5**	**-1**
Node sets	{2,3,4},{0,1,5,6},{7}							
Selected edge	{4-2}							

Step 6: Select edge 0-7 between Nodes



i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Parent[*i*]	**5**	**5**	**4**	**4**	**-3**	**-4**	**5**	**0**
Node sets	{2,3,4},{0,1,5,6,7}							
Selected edge	{0-7}							

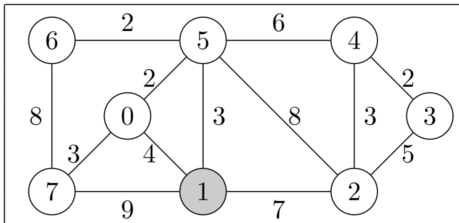
Step 7: Select edge 5-4 between Nodes



i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Parent[*i*]	**5**	**5**	**4**	**4**	**5**	**-5**	**5**	**0**
Node sets	{0,1,2,3,4,5,6,7}							
Selected edge	{5-4}							

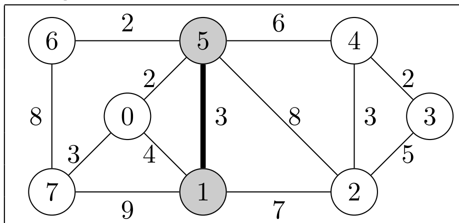
b. Prim's

Stage 1



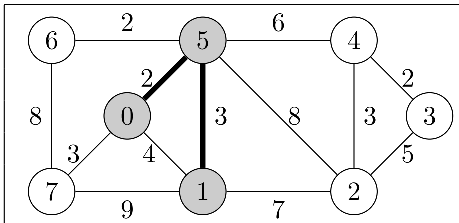
i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Nearest[*i*]	**4**	**0**	**7**	∞	∞	**3**	∞	**9**
Parent[*i*]	**1**	**-1**	**1**	**-**	**-**	**1**	**-**	**1**
InTheTree[*i*]	**F**	**T**	**F**	**F**	**F**	**F**	**F**	**F**
Weight	0							

Stage 2



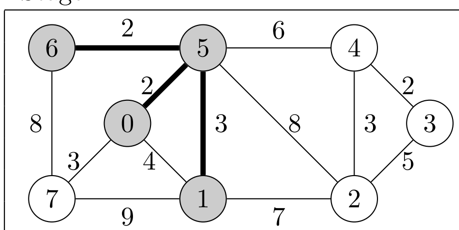
i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Nearest[*i*]	**2**	**0**	**7**	∞	**6**	**3**	**2**	**9**
Parent[*i*]	**5**	**-1**	**1**	**-**	**5**	**1**	**5**	**1**
InTheTree[*i*]	**F**	**T**	**F**	**F**	**F**	**T**	**F**	**F**
Weight	3							

Stage 3



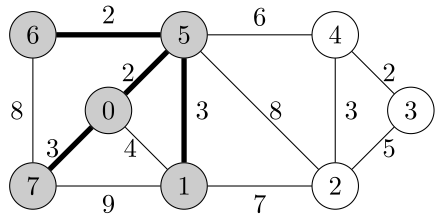
i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Nearest[*i*]	**2**	**0**	**7**	∞	**6**	**3**	**2**	**3**
Parent[*i*]	**5**	**-1**	**1**	**-**	**5**	**1**	**5**	**0**
InTheTree[*i*]	**T**	**T**	**F**	**F**	**F**	**T**	**F**	**F**
Weight	5							

Stage 4



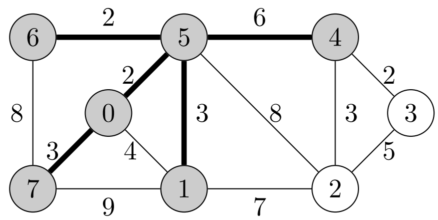
i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Nearest[*i*]	**2**	**0**	**7**	∞	**6**	**3**	**2**	**3**
Parent[*i*]	**5**	**-1**	**1**	**-**	**5**	**1**	**5**	**0**
InTheTree[*i*]	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**F**
Weight	7							

Stage 5



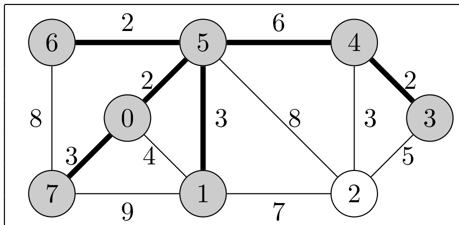
i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
$Nearest[i]$	**2**	**0**	**7**	∞	**6**	**3**	**2**	**3**
$Parent[i]$	**5**	**-1**	**1**	**-**	**5**	**1**	**5**	**0**
$InTheTree[i]$	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**
$Weight$	10							

Stage 6



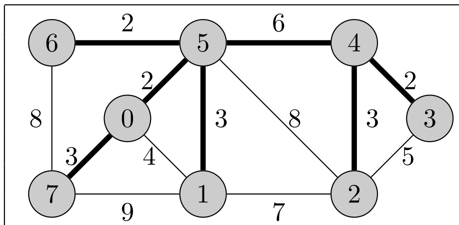
i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
$Nearest[i]$	**2**	**0**	**3**	**2**	**6**	**3**	**2**	**3**
$Parent[i]$	**5**	**-1**	**4**	**4**	**5**	**1**	**5**	**0**
$InTheTree[i]$	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**T**
$Weight$	16							

Stage 7



i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
$Nearest[i]$	**2**	**0**	**3**	**2**	**6**	**3**	**2**	**3**
$Parent[i]$	**5**	**-1**	**4**	**4**	**5**	**1**	**5**	**0**
$InTheTree[i]$	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**
$Weight$	18							

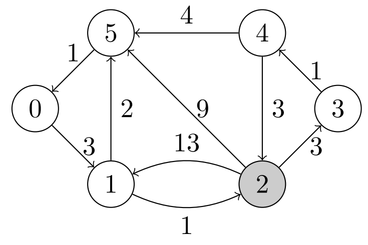
Final



After stage 7, array *Parent* is now complete and implements a minimum spanning tree with weight 21.

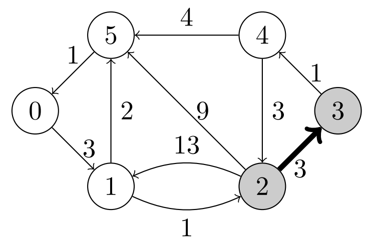
4 Trace the action of procedure *Dijkstra* for the following digraph with initial vertex $r = 2$.

Stage 1



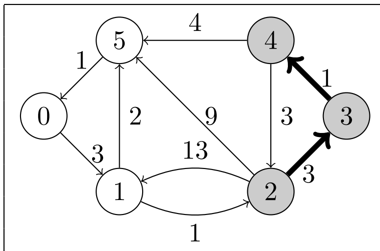
i	**0**	**1**	**2**	**3**	**4**	**5**
$Dist[i]$	∞	**13**	**0**	**3**	∞	**9**
$Parent[i]$	**-**	**2**	**-1**	**2**	**-**	**2**
$InTheTree[i]$	**F**	**F**	**T**	**F**	**F**	**F**
$Total\ Distance$	0					

Stage 2



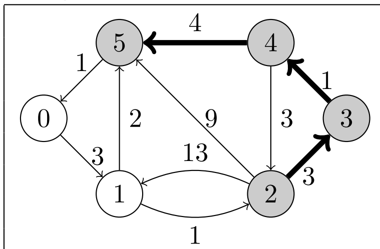
i	**0**	**1**	**2**	**3**	**4**	**5**
$Dist[i]$	∞	**13**	**0**	**3**	**4**	**9**
$Parent[i]$	**-**	**2**	**-1**	**2**	**3**	**2**
$InTheTree[i]$	**F**	**F**	**T**	**T**	**F**	**F**
$Total\ Distance$	3					

Stage 3



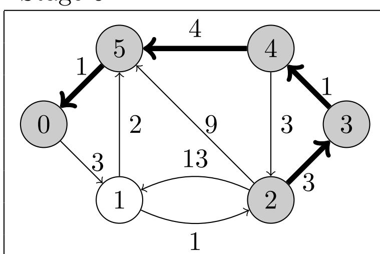
i	**0**	**1**	**2**	**3**	**4**	**5**
$Dist[i]$	∞	**13**	**0**	**3**	**4**	**8**
$Parent[i]$	**-**	**2**	**-1**	**2**	**3**	**4**
$InTheTree[i]$	**F**	**F**	**T**	**T**	**T**	**F**
$Total\ Distance$	4					

Stage 4



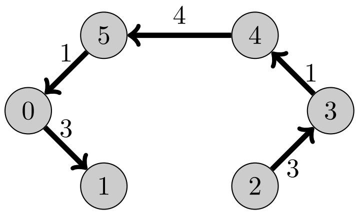
i	**0**	**1**	**2**	**3**	**4**	**5**
$Dist[i]$	**9**	**13**	**0**	**3**	**4**	**8**
$Parent[i]$	**5**	**2**	**-1**	**2**	**3**	**4**
$InTheTree[i]$	**F**	**F**	**T**	**T**	**T**	**T**
$Total\ Distance$	8					

Stage 5



i	**0**	**1**	**2**	**3**	**4**	**5**
$Dist[i]$	**9**	**12**	**0**	**3**	**4**	**8**
$Parent[i]$	**5**	**0**	**-1**	**2**	**3**	**4**
$InTheTree[i]$	**T**	**F**	**T**	**T**	**T**	**T**
$Total\ Distance$	9					

Because Dijkstra's algorithm terminates after only $n - 1$ stages, the algorithm is complete and we add *Node* 1 for a final cost/distance of 12.



5 For $C = 30$, we can instantly discard $i = 1$ since its weight is > 30. Then, we'll add $i = 2, 3, 4, 6$ for $w = 29$ and $v = 95$. That leaves $i = 0, 7$ as options, and to choose the higher value, we chose $\frac{1}{30}th$ of $i = 0$ for a $v = 2$. We've met our capacity with a final value of $v = 97$.

6 Design and analyze an algorithm $MultInt$ for multiplying large integers.

The problem of dealing with arbitrarily large polynomials is similar to the problem of arbitrarily large integers. Thus, assuming that the integers have the same number of digits, we can base our algorithm on $PolyMult1$ and instead of splitting polynomials, we will just split the integer, and instead of multiplying by x, we use our base-10. Therefore, $PolyMult1$ will be transformed into $MultInt$ with integers $A = a_1 + a_2 10^d +, B = b_1 + b_2 10^d$ and the analysis is also the same as $PolyMult1$ with $W(n) = \Theta(n^{log_2 3})$

function $MultInt(A, B, n)$ **recursive**
Input: A=LargeInt, B=LargeInt, n=positive int (length of A,B)
Output: AB (product)
  if $n = 1$ then
    return(AB)
  else
    $d \leftarrow \lceil n/2 \rceil$
    $Split(A, a_1, a_2)$
    $Split(B, b_1, b_2)$
    $C \leftarrow MultInt(a_2, b_2, d)$
    $D \leftarrow MultInt(a_1 + a_2, b_1 + b_2, d)$
    $E \leftarrow MultInt(a_1, b_1, d)$
    return$(10^{2d}C + 10^d(D - C - E) + E)$
  end if
end $MultInt$

7 Verify Proposition 8.4.4.
To verify this, we need the following to hold:

$$AB = \begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix} \begin{bmatrix} b_{00} & b_{01} \\ b_{10} & b_{11} \end{bmatrix}$$

$$= \begin{bmatrix} a_{00}b_{00} + a_{01}b_{10} & a_{00}b_{01} + a_{01}b_{11} \\ a_{10}b_{00} + a_{11}b_{10} & a_{10}b_{01} + a_{11}b_{11} \end{bmatrix} = \begin{bmatrix} m_1 + m_4 - m_5 + m_7 & m_3 + m_5 \\ m_2 + m_4 & m_1 + m_3 - m_2 + m_6 \end{bmatrix}$$

Where m_x is defined in the following:
$m_1 = (a_{00} + a_{11})(b_{00} + b_{11})$
$m_2 = (a_{10} + a_{11})(b_{00})$
$m_3 = (a_{00})(b_{01} - b_{11})$
$m_4 = (a_{11})(b_{10} - b_{00})$
$m_5 = (a_{00} + a_{01})(b_{11})$
$m_6 = (a_{10} - a_{00})(b_{00} + b_{01})$
$m_7 = (a_{01} - a_{11})(b_{10} + b_{11})$
Thus, expanding the last matrix, we have:

$$\begin{aligned} m_1 + m_4 - m_5 + m_7 &= (a_{00} + a_{11})(b_{00} + b_{11}) + (a_{11})(b_{10} - b_{00}) - (a_{00} + a_{01})(b_{11}) + (a_{01} - a_{11})(b_{10} + b_{11}) \\ &= a_{00}b_{00} + a_{00}b_{11} + a_{11}b_{00} + a_{11}b_{11} + a_{11}b_{10} + a_{01}b_{11} + a_{01}b_{10} \\ &\qquad - a_{00}b_{11} - a_{11}b_{00} - a_{11}b_{11} - a_{11}b_{10} - a_{01}b_{11} \\ &= a_{00}b_{00} + a_{01}b_{10} \end{aligned}$$

$$\begin{aligned} m_3 + m5 &= (a_{00})(b_{01} - b_{11}) + (a_{00} + a_{01})(b_{11}) \\ &= a_{00}b_{01} - a_{00}b_{11} + a_{00}b_{11} + a_{01}b_{11} \\ &= a_{00}b_{01} + a_{01}b_{11} \end{aligned}$$

$$\begin{aligned} m_2 + m_4 &= (a_{10} + a_{11})(b_{00}) + (a_{11})(b_{10} - b_{00}) \\ &= a_{01}b_{00} + a_{11}b_{00} + a_{11}b_{10} - a_{11}b_{00} \\ &= a_{01}b_{00} + a_{11}b_{10} \end{aligned}$$

$$\begin{aligned} m_1 + m_3 - m_2 + m_6 &= (a_{00} + a_{11})(b_{00} + b_{11}) + (a_{00})(b_{01} - b_{11}) - (a_{10} + a_{11})(b_{00}) + (a_{10} - a_{00})(b_{00} + b_{01}) \\ &= a_{00}b_{00} + a_{00}b_{11} + a_{11}b_{00} + a_{11}b_{11} + a_{00}b_{01} + a_{10}b_{00} + a_{10}b_{01} \\ &\quad -a_{00}b_{00} - a_{00}b_{11} - a_{11}b_{00} \qquad - a_{00}b_{01} - a_{10}b_{00} \\ &= a_{10}b_{01} + a_{11}b_{11} \end{aligned}$$

8

$M_1 = (A_{00} + A_{11})(B_{00} + B_{11}) = \begin{bmatrix} 7 & 7 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 4 & 1 \end{bmatrix}$
$m_1 = (a_{00} + a_{11})(b_{00} + b_{11}) = 10 \times 6 = 60$
$m_2 = (a_{10} + a_{11})(b_{00}) = 8 \times 5 = 40$
$m_3 = (a_{00})(b_{01} - b_{11}) = 7 \times 1 = 7$
$m_4 = (a_{11})(b_{10} - b_{00}) = 3 \times (-1) = -3$
$m_5 = (a_{00} + a_{01})(b_{11}) = 14 \times 1 = 14$
$m_6 = (a_{10} - a_{00})(b_{00} + b_{01}) = (-2) \times 7 = -14$
$m_7 = (a_{01} - a_{11})(b_{10} + b_{11}) = 4 \times 5 = 20$
$\boxed{M_1} = \begin{bmatrix} m_1 + m_4 - m_5 + m_7 & m_3 + m_5 \\ m_2 + m_4 & m_1 + m_3 - m_2 + m_6 \end{bmatrix} = \begin{bmatrix} 63 & 21 \\ 37 & 13 \end{bmatrix}$

$M_2 = (A_{10} + A_{11})(B_{00}) = \begin{bmatrix} 8 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}$
$m_1 = (a_{00} + a_{11})(b_{00} + b_{11}) = 12$
$m_2 = (a_{10} + a_{11})(b_{00}) = 0$
$m_3 = (a_{00})(b_{01} - b_{11}) = 8$
$m_4 = (a_{11})(b_{10} - b_{00}) = 0$
$m_5 = (a_{00} + a_{01})(b_{11}) = 11$
$m_6 = (a_{10} - a_{00})(b_{00} + b_{01}) = -12$
$m_7 = (a_{01} - a_{11})(b_{10} + b_{11}) = 1$
$\boxed{M_2} = \begin{bmatrix} m_1 + m_4 - m_5 + m_7 & m_3 + m_5 \\ m_2 + m_4 & m_1 + m_3 - m_2 + m_6 \end{bmatrix} = \begin{bmatrix} 0 & 19 \\ 0 & 8 \end{bmatrix}$

$M_3 = (A_{00})(B_{01} - B_{11}) = \begin{bmatrix} 2 & 0 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} -4 & 1 \\ -4 & -1 \end{bmatrix}$
$m_1 = (a_{00} + a_{11})(b_{00} + b_{11}) = -10$
$m_2 = (a_{10} + a_{11})(b_{00}) = -12$
$m_3 = (a_{00})(b_{01} - b_{11}) = 4$
$m_4 = (a_{11})(b_{10} - b_{00}) = 0$
$m_5 = (a_{00} + a_{01})(b_{11}) = -2$
$m_6 = (a_{10} - a_{00})(b_{00} + b_{01}) = -3$
$m_7 = (a_{01} - a_{11})(b_{10} + b_{11}) = 0$
$\boxed{M_3} = \begin{bmatrix} m_1 + m_4 - m_5 + m_7 & m_3 + m_5 \\ m_2 + m_4 & m_1 + m_3 - m_2 + m_6 \end{bmatrix} = \begin{bmatrix} -8 & 2 \\ -12 & 3 \end{bmatrix}$

$M_4 = (A_{11})(B_{10} - B_{00}) = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 6 & 0 \end{bmatrix}$
$m_1 = (a_{00} + a_{11})(b_{00} + b_{11}) = 24$
$m_2 = (a_{10} + a_{11})(b_{00}) = 15$
$m_3 = (a_{00})(b_{01} - b_{11}) = -10$
$m_4 = (a_{11})(b_{10} - b_{00}) = 9$
$m_5 = (a_{00} + a_{01})(b_{11}) = 0$
$m_6 = (a_{10} - a_{00})(b_{00} + b_{01}) = -3$
$m_7 = (a_{01} - a_{11})(b_{10} + b_{11}) = 24$
$\boxed{M_4} = \begin{bmatrix} m_1 + m_4 - m_5 + m_7 & m_3 + m_5 \\ m_2 + m_4 & m_1 + m_3 - m_2 + m_6 \end{bmatrix} = \begin{bmatrix} 57 & -10 \\ 24 & -4 \end{bmatrix}$

$M_5 = (A_{00} + A_{01})(B_{11}) = \begin{bmatrix} 3 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 4 & 0 \end{bmatrix}$
$m_1 = (a_{00} + a_{11})(b_{00} + b_{11}) = 35$
$m_2 = (a_{10} + a_{11})(b_{00}) = 35$
$m_3 = (a_{00})(b_{01} - b_{11}) = 0$

$m_4 = (a_{11})(b_{10} - b_{00}) = -4$
$m_5 = (a_{00} + a_{01})(b_{11}) = 0$
$m_6 = (a_{10} - a_{00})(b_{00} + b_{01}) = 0$
$m_7 = (a_{01} - a_{11})(b_{10} + b_{11}) = -12$

$$\boxed{M_5} = \begin{bmatrix} m_1 + m_4 - m_5 + m_7 & m_3 + m_5 \\ m_2 + m_4 & m_1 + m_3 - m_2 + m_6 \end{bmatrix} = \begin{bmatrix} 19 & 0 \\ 31 & 0 \end{bmatrix}$$

$$M_6 = (A_{10} - A_{00})(B_{00} + B_{01}) = \begin{bmatrix} 1 & -4 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}$$

$m_1 = (a_{00} + a_{11})(b_{00} + b_{11}) = 2$
$m_2 = (a_{10} + a_{11})(b_{00}) = -2$
$m_3 = (a_{00})(b_{01} - b_{11}) = 3$
$m_4 = (a_{11})(b_{10} - b_{00}) = -1$
$m_5 = (a_{00} + a_{01})(b_{11}) = 0$
$m_6 = (a_{10} - a_{00})(b_{00} + b_{01}) = -16$
$m_7 = (a_{01} - a_{11})(b_{10} + b_{11}) = 0$

$$\boxed{M_6} = \begin{bmatrix} m_1 + m_4 - m_5 + m_7 & m_3 + m_5 \\ m_2 + m_4 & m_1 + m_3 - m_2 + m_6 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ -3 & -9 \end{bmatrix}$$

$$M_7 = (A_{01} - A_{11})(B_{10} + B_{11}) = \begin{bmatrix} -4 & -6 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 8 & 0 \\ 10 & 1 \end{bmatrix}$$

$m_1 = (a_{00} + a_{11})(b_{00} + b_{11}) = -27$
$m_2 = (a_{10} + a_{11})(b_{00}) = -8$
$m_3 = (a_{00})(b_{01} - b_{11}) = 4$
$m_4 = (a_{11})(b_{10} - b_{00}) = 2$
$m_5 = (a_{00} + a_{01})(b_{11}) = -10$
$m_6 = (a_{10} - a_{00})(b_{00} + b_{01}) = 16$
$m_7 = (a_{01} - a_{11})(b_{10} + b_{11}) = 77$

$$\boxed{M_7} = \begin{bmatrix} m_1 + m_4 - m_5 + m_7 & m_3 + m_5 \\ m_2 + m_4 & m_1 + m_3 - m_2 + m_6 \end{bmatrix} = \begin{bmatrix} -92 & -6 \\ -6 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} M_1 + M_4 - M_5 + M_7 & M_3 + M_5 \\ M_2 + M_4 & M_1 + M_3 - M_2 + M_6 \end{bmatrix}$$

$$= \begin{bmatrix} (63 + 57 - 19 - 92) & (21 - 10 - 6) & (-8 + 19) & (2 + 0) \\ (37 + 24 - 31 - 6) & (13 - 4 + 1) & (-12 + 31) & (3 + 0) \\ (0 + 57) & (19 - 10) & (63 - 8 + 0 + 1) & (21 + 2 - 19 + 3) \\ (0 + 24) & (8 - 4) & (37 - 12 - 0 - 3) & (13 + 3 - 8 - 9) \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 5 & 11 & 2 \\ 24 & 10 & 19 & 3 \\ 57 & 9 & 56 & 7 \\ 24 & 4 & 22 & -1 \end{bmatrix}$$